J. Fluid Mech. (1967), *vol.* 27, *part* 2, *pp.* 353–360
Printed in Great Britain

The formation of elliptical wakes

By YUNG-HUANG KUO† AND LIONEL V. BALDWIN
Fluid Dynamics and Diffusion Laboratory, College of Engineering,
Colorado State University, Fort Collins, Colorado

(Received 27 August 1965 and in revised form 18 May 1966)

The wakes formed behind sharp-edged, bluff, elliptical bodies have elliptical cross-sections, but the major axis of the wake is aligned with the minor axis of the body. This effect was observed in both mean velocity and turbulent intensity data in wakes throughout the range of the experiment, from several body diameters to distances of 250 diameters downstream of the body. The turbulent energy in the wake flow near the body displayed a periodicity which was correlated using a Strouhal number. Over the Reynolds-number range from 8×10^3 to 7×10^4, the Strouhal number depended only on the body eccentricity.

1. Introduction

An experimental study of turbulent, elliptical wakes offers several advantages which are not found in the axisymmetric or two-dimensional geometries to test eddy-diffusivity approximations. The present work was undertaken to provide experimental data for comparison with theoretical predictions previously published by Steiger & Bloom (1963) and the authors (1964) for the far wake region where an Oseen approximation may be valid. A complete description of this research is available in the dissertation of Kuo (1965).

This note summarizes an unexpected observation: wakes formed behind sharp-edged, bluff, elliptical bodies have elliptical sections, but the major axis of the wake several body diameters downstream of the body is aligned with the minor axis of the body. This discovery was 'unexpected' because it had previously not been predicted theoretically nor observed experimentally (e.g. see the reviews of Rosenhead 1953; Marris 1964; or Halleen 1964).

2. Experimental arrangement

The experiments were conducted in a low-speed wind tunnel located in the Fluid Dynamics and Diffusion Laboratory of Colorado State University. The tunnel is a recirculating type, and has a test section 6 ft. × 6 ft. in cross-section and 30 ft. long. An inlet contraction ratio of 4 to 1 with damping screens yields a free-stream turbulence level of approximately $\frac{1}{2}$ %. The mean velocity U_∞ in undisturbed flow was set at three values (58·3, 29·8 and 19·6 ft./sec) in the course of the experiments.

Figure 1 is a sketch showing the disks mounted in the inlet of the test section

† Present address: Department of Civil Engineering, Louisiana State University, Baton Rouge.

of the wind tunnel. This sketch also defines the co-ordinates for the wake survey and depicts the orientation of the mean velocity wake.

Table 1 lists the dimensions of the disks which were used to generate the wakes. One set had an equivalent frontal area of a 1 in. circular disk, while the other set had the area of a 3 in. circular disk. Each set contained a circular disk and two elliptical disks having eccentricities ϵ (ratio of minor to major axis) of 0·6 and 0·2. The disks were made of plastic and had a sharp edge on the upstream face.

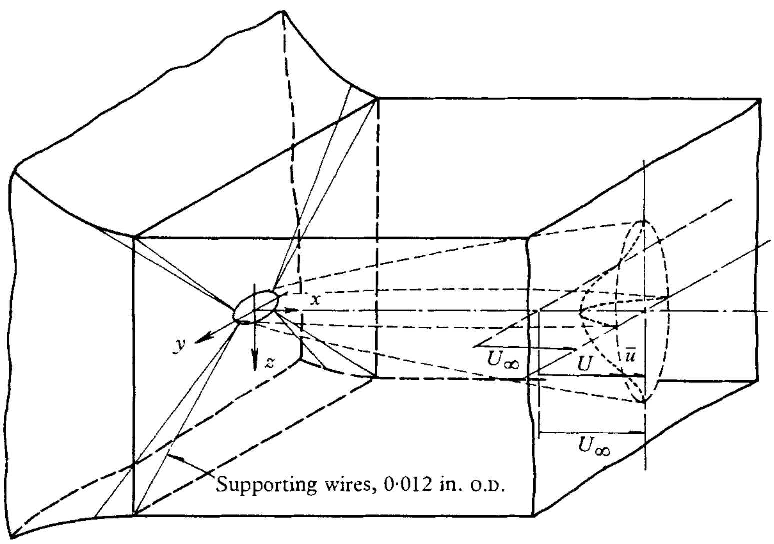


FIGURE 1. Sketch showing position of the disk in the wind tunnel.

Body eccentricity	Equivalent diameter of disk	Major axis (in.)	Minor axis (in.)	Thickness of disks (in.)
1·0	3 in.	3·00	3·00	0·50
	1 in.	1·00	1·00	0·25
0·6	3 in. equivalent	3·88	2·32	0·50
	1 in. equivalent	1·29	0·775	0·25
0·2	3 in. equivalent	6·72	1·34	0·50
	1 in. equivalent	2·24	0·448	0·25

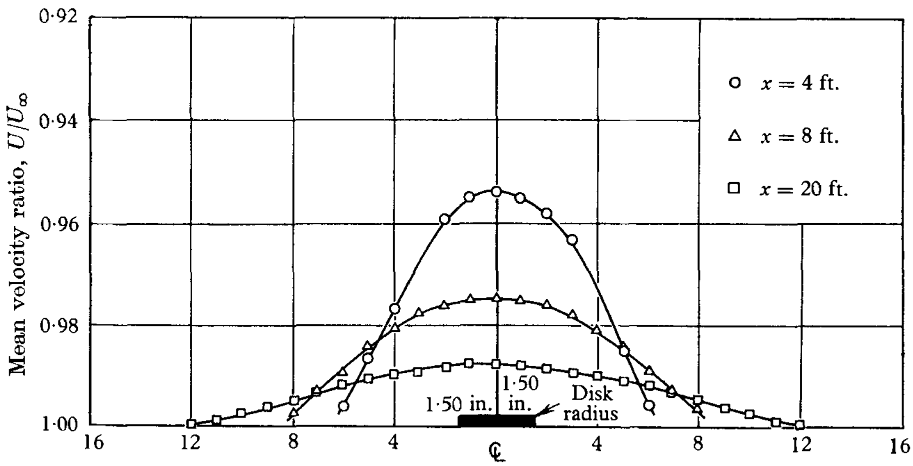
TABLE 1. Dimensions of disks.

The mean velocity was calculated from Pitot tube data. A constant-temperature, hot-wire anemometer was used to sense the turbulent fluctuations in axial velocity. A spectral analysis of the anemometer signal was performed using two analogue circuits: a constant-percentage, band-pass analyser (Bruel and Kjaer Type 2109) and a constant band-pass (± 1 c/s) analyser (Technical Products Spectrum Analyser TP 626, 627, 633).

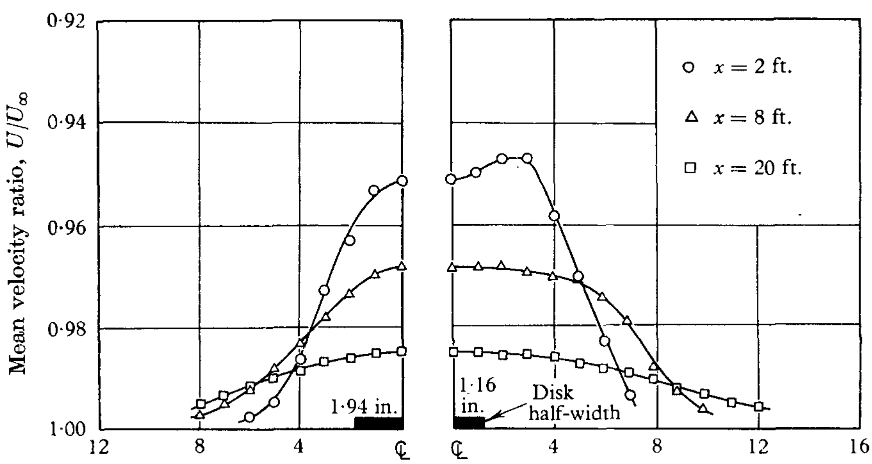
3. Results and discussion

The configuration of the elliptical wake is illustrated in typical mean velocity profiles shown in figure 2. The radii or half-widths of the disks which formed the wakes are shown as solid bars on the abscissa. The larger wake dimension is aligned with the smaller body dimension. Similar data were obtained for 1 in. equivalent diameter disks having the same eccentricity range. For the data shown in figure 2, the Reynolds number based on free-stream velocity U_∞ and equivalent body diameter D is $7{\cdot}1 \times 10^4$. The circular disks generated axially-symmetric wakes; this fact was verified by surveys not reproduced in figure 2.

Tests showed that the wake formation relative to the disk was unaffected by changes in the disk-mounting system and in the angular orientation of the disk relative to the tunnel walls. That is, a diagonal mount utilizing four wires in the plane of the disk (rather than eight wires, axially offset as in figure 1) caused no appreciable change in wake alignment or growth rate, even though the disk vibrated noticeably in this arrangement under test flows. When an elliptical disk was suspended with its major axis aligned vertically (opposite to data cited

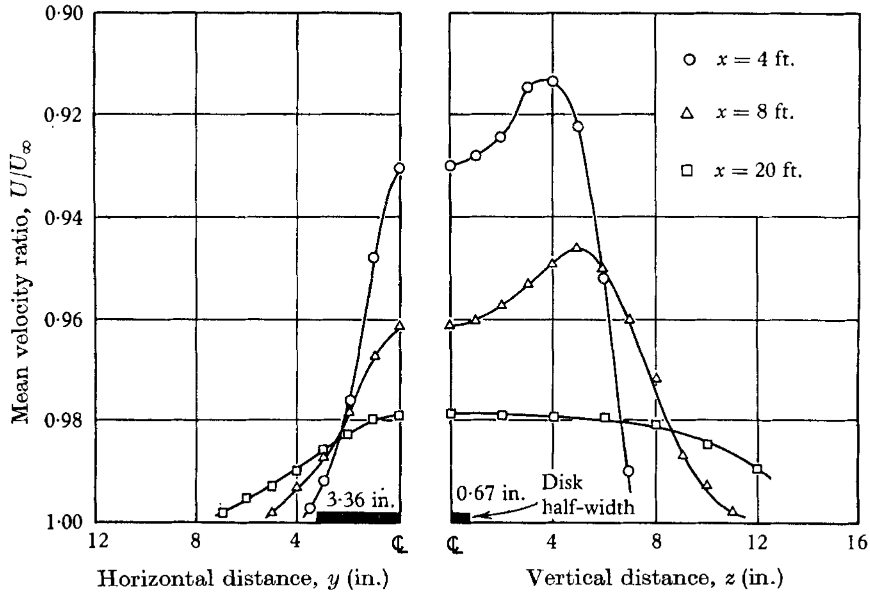


(*a*)



(*b*)

FIGURE 2 *a*, *b*. For legend see next page.

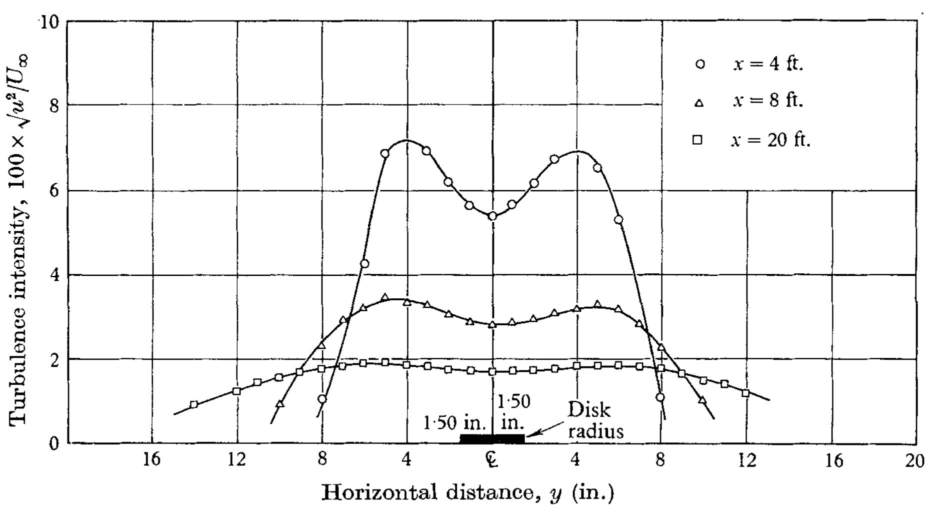


(*c*)

FIGURE 2. Velocity profiles at various axial stations behind disks having 3 in. equivalent diameter in ambient air flow. $U_\infty = 58{\cdot}3$ ft./sec. (*a*) Circular disk, $\epsilon = 1{\cdot}0$; (*b*) elliptical disk, $\epsilon = 0{\cdot}6$; (*c*) elliptical disk, $\epsilon = 0{\cdot}2$.

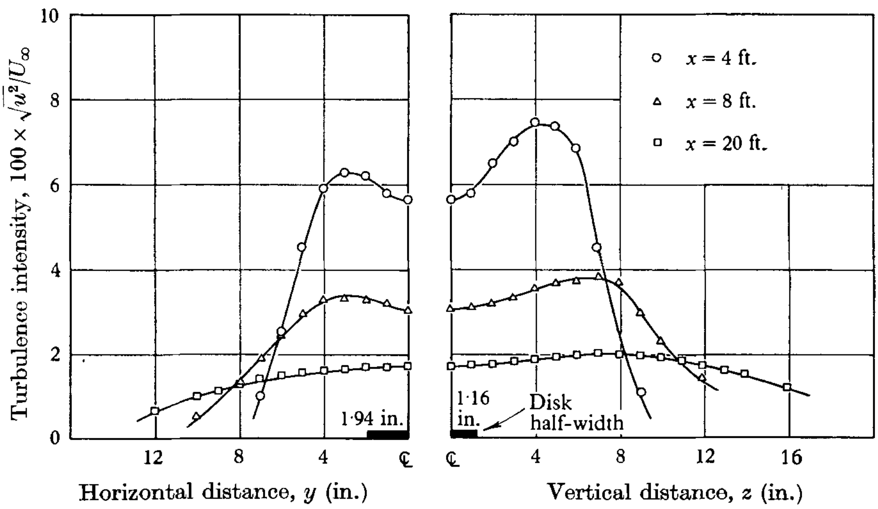
herein), the wake sections showed the minor axis of the wake to be aligned vertically.

Hot-wire anemometer surveys showed that the turbulence intensity profiles of the wake were also aligned in a manner analogous to the mean velocity profiles. Typical profiles are shown in figure 3 where the radii or half-widths of the disks

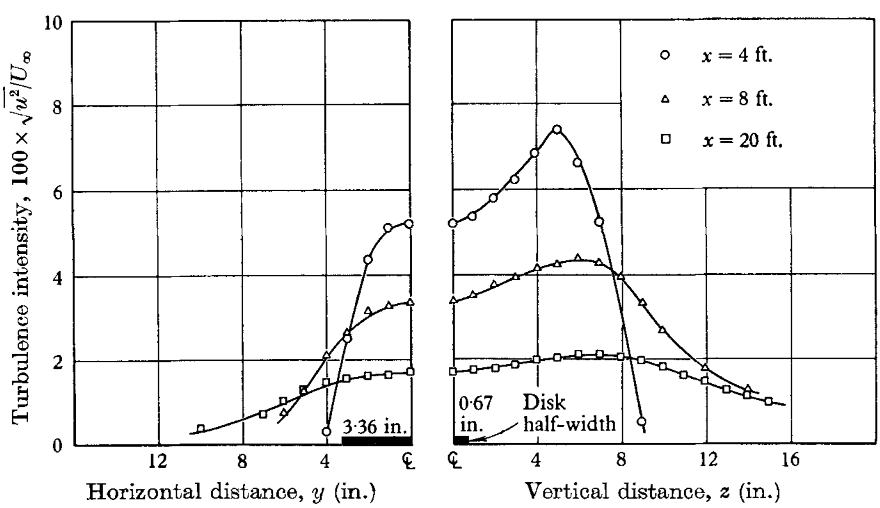


(*a*)

FIGURE 3*a*. For legend see facing page.



(*b*)



(*c*)

FIGURE 3. Turbulence intensity profiles at various axial stations in ambient air flow. $U_\infty = 58{\cdot}3$ ft./sec. (*a*) 3 in. diameter, circular disk; (*b*) 3 in. equivalent diameter, elliptical disk, $\epsilon = 0{\cdot}6$; (*c*) 3 in. equivalent diameter, elliptical disk, $\epsilon = 0{\cdot}2$.

which formed the wakes are shown as solid bars on the abscissa. The larger wake dimension is associated with the smaller body dimension.

Immediately behind the body, the wake spreads more rapidly in the direction of the short axis than in the direction of the long axis. This interchange of size persists throughout the downstream region. In fact, the elliptical wakes tend to become axisymmetric far downstream as predicted by Steiger & Bloom (1963). The wake phenomenon reported here differs appreciably in this respect from the 'overshooting' of jets from rectangular apertures.

To study the formation of the wake in more detail, a single hot-wire anemometer was placed 1·5 ft. downstream of each disk. A survey was made with the anemometer which passed through the centreline of the wake on a path parallel to the minor axis of the disk. At the maximum in the turbulent intensity profile, a power spectral analysis of the axial component of the turbulence was performed. This maximum intensity occurs near the edge of the wake; for example, examine the profiles at $x/D = 4$ in figure 3. Sample spectra are shown in figure 4 for three

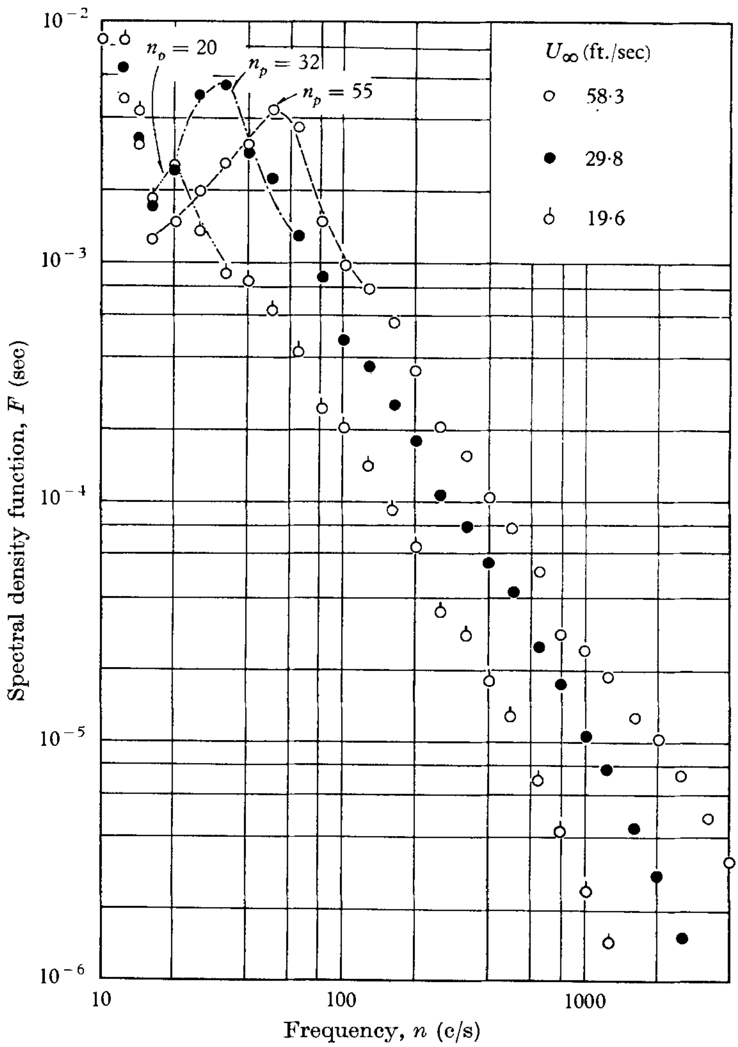


FIGURE 4. Power spectral density function for turbulence in wake of elliptical 3 in. diameter equivalent disk ($\epsilon = 0·2$, $x = 1·5$ ft.).

mean velocities. Note the discrete peak in the turbulent energy at a frequency n_p which is a function of mean velocity. This peak is associated with the periodicity of the vortices shed at the edge of the disk. The periodicity in the turbulent flow was correlated using a Strouhal number based on the frequency of the peak turbulent energy, free stream velocity and equivalent body diameter. Figure 5 shows that over the Reynolds number range of this experiment, the Strouhal number depends only on the body eccentricity; $S = 0{\cdot}145$ for the circular disks, $S = 0{\cdot}168$ for $\epsilon = 0{\cdot}6$ and $S = 0{\cdot}237$ for $\epsilon = 0{\cdot}2$. Extrapolation of this trend to an eccentricity of $\epsilon = 0$, which corresponds to a long flat plate, is not possible. For this case, Roshko (1955) found $S = 0{\cdot}14$. The complexity of the wake formation is illustrated by the

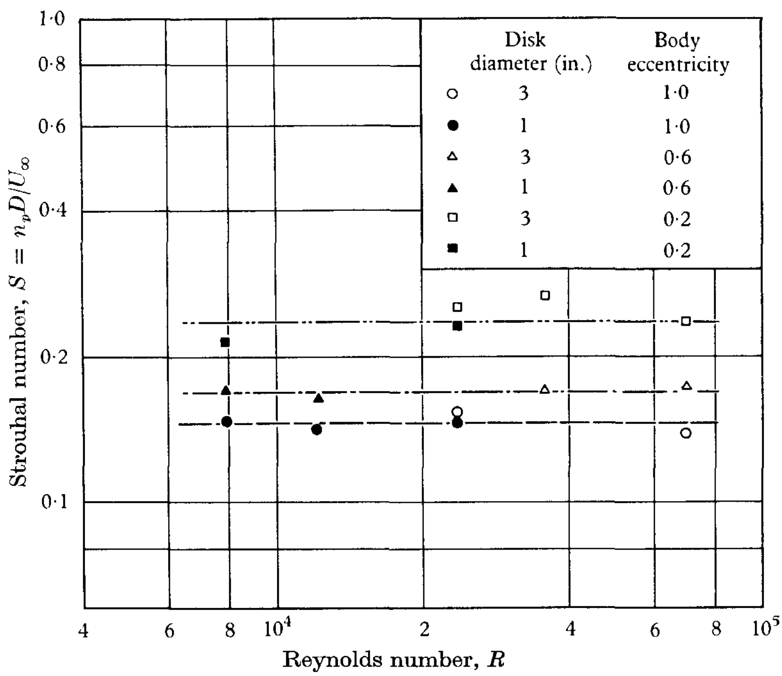


FIGURE 5. Correlation of Strouhal number data in the wake of bluff, elliptical bodies.

fact that there exists a second predominant vortex shedding frequency associated with the small side of the disk. When a hot-wire survey was made parallel to the major axis of the elliptical disk and a power spectral density analysis made of the maximum signal at the same axial station ($x = 1{\cdot}5$ ft.), a peak in the turbulent energy distribution was observed which correlated with $S = 0{\cdot}124$ for the $\epsilon = 0{\cdot}2$ disk.

4. Flow visualization

Photographs and moving pictures were taken of the flow behind elliptical disks in an open-channel flow of water at 0·15 ft./sec. The wake was made visible by hydrogen bubbles generated at a wire immediately upstream of the disk. The pictures are not reproduced herein. However, a brief moving picture summarizing the preliminary study is available for loan from the second author (L. V. B.). The movie confirms that the growth angle is markedly different for the two axes of the elliptical wakes and that the dominant vorticity is generated along the long axis of the disk in alternately shedding vortices. This flow-visualization technique is a powerful tool (e.g. Schraub *et al.* 1964), and it was not fully exploited in these exploratory studies.

Although certain aspects of the formation of wakes behind elliptical disks have been discussed here, we have avoided presenting an hypothesis of 'why' the wake grows non-uniformly behind the bluff body. It is not difficult to speculate, but it seems prudent to study, the base pressure and a more complete set of movie visualizations before proposing a model.

This research was sponsored by the Bureau of Ships Fundamental Hydromechanics Research Programme administered by the David Taylor Model Basin under Nonr 1610(08).

REFERENCES

HALLEEN, R. M. 1964 A literature review on subsonic free turbulent shear flow. *Stanford University Rept.* no. MD 11 (*AFOSR*-TN-5444).

KUO, Y. H. 1965 Three-dimensional turbulent wake. A dissertation presented in partial fulfillment of requirements for the degree Doctor of Philosophy at Colorado State University.

KUO, Y. H. & BALDWIN, L. V. 1964 Comments on three-dimensional effects in viscous wakes. *AIAA J.* **2**, 1163–4.

MARRIS, A. W. 1964 A review on vortex streets, periodic wakes and induced vibration phenomena. *Trans. ASME* (Series D). *J. Basic Eng.* **86**, 185–196.

ROSENHEAD, L. 1953 *Vortex Systems in Wakes. Advances in Applied Mechanics*, vol. III. New York: Academic Press.

ROSHKO, A. 1955 On the wake and drag of bluff bodies. *J. Aero. Sci.* **22**, 124–32.

SCHRAUB, F. A., KLINE, S. J., HENRY, J., RUNSTADLER, T. W. & LITTELL, A. 1964 Use of hydrogen bubbles for quantitative determination of time dependent velocity fields in low speed water flow. *Stanford Univ. Rept.* no. MD 10. (*AFOSR* Contract AF 49(638)–1278.)

STEIGER, M. H. & BLOOM, M. H. 1963 Three-dimensional effects in viscous wakes. *AIAA J.* **1**, 776–82.